Non-local scaling in two–dimensional gravitational clustering

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ABSTRACT

Using an ensemble of high resolution 2D numerical simulations, we explore the scaling properties of cosmological density fluctuations in the non-linear regime. We study the scaling behaviour of the usual N–point volume-averaged correlations, but also examine the scaling of the entire probability density function (pdf) of the fluctuations. We focus on two important issues: (i) whether the scaling behaviour of 2D clustering is consistent with what one infer from radial collapse arguments; and (ii) whether there is any evidence from these high-resolution simulations that a regime of *stable clustering* is ever entered. We find that the answers are (i) yes and (ii) no. We further find that the behaviour of the highly non-linear regime in these simulations suggests the existence of a regime where the correlation function is *independent* of the initial power spectrum.

Key words: Cosmology: theory – large-scale structure of the Universe – Methods: analytical

1 INTRODUCTION

The standard model of cosmological structure formation is based on the idea that small initial density fluctuations grew by the action of gravitational instability into the large inhomogeneities we observe today. While the initial, linear stages of fluctuation growth have been understood analytically for many years, the later stages are less amenable to analytical study because these are described by non-linear gravitational physics involved (e.g. Sahni & Coles 1995). The standard approach for the early stages is via perturbation theory, where an expansion in an appropriately chosen small parameter is employed (e.g. Fry 1984; Moutarde et al. 1991; Buchert 1992; Bernardeau 1992; Munshi et al. 1997). Such an expansion is typically taken to first order when the density fluctuations are very small in amplitude, but some notable successes have been achieved in the weakly non-linear regime when the density fluctuations are of order unity, by taking perturbation theory to higher orders. On the other hand, the strongly non-linear regime where the density fluctuations exceed unity by a large factor is intrinsically non-linear and a conceptually different approach is required. One method involves adopting the so-called *stable clustering* ansatz (e.g. Peebles 1980), but this ansatz only applies (if at all) when density contrasts are very large indeed (Jain 1997). Probing the regime leading up to the stable clustering limit is considerably more difficult, but some progress has been made using arguments based on the properties of spherical collapse (Hamilton et al. 1991; Nityananda & Padmanabhan 1994; Jain, Mo & White 1995; Padmanabhan 1996; Padmanabhan et al. 1996; Peacock & Dodds 1996). These arguments allow a simple scaling model to be constructed, which can describe certain aspects of the statistical behaviour of density fluctuations all the way from the linear regime to the stable clustering limit by a kind of functional interpolation between the linear and stable clustering limits, guided by the behaviour of numerical simulations. Since the original paper by Hamilton et al. (1991), there has been an exploration of the physical origin of this functional behaviour (Nityanada & Padmanabhan 1994; Padmanabhan 1996; Padmanabhan et al. 1996; Munshi & Padmanabhan 1997), and some generalisations of the original arguments have been presented (e.g. Jain, Mo & White 1995; Peacock & Dodds 1996).

In this paper we address two issues related to these scaling arguments, and the models emerging from them. The first concerns the intermediate regime. We argue that the behaviour of correlation functions in this regime is basically governed by the number of spatial dimensions. We confirm this argument using a battery of two-dimensional N-body experiments, and find the results to be consistent with our assertion, lending theoretical understanding to the numerical fits obtained from three-dimensional simulations. An additional advantage of using two-dimensional simulations is that it is possible to obtain high resolution at relatively small computational cost compared to full three-dimensional sim-

ulations. With this in mind we also address the (related) issue of the stable clustering limit itself, and whether there is any evidence from our simulations that this description is appropriate at all during the highly non-linear stages.

2 SCALING IN GRAVITATIONAL CLUSTERING

The evolution of the volume-averaged two-point correlation function $\bar{\xi}_2(x,a)$ in a D-dimensional space can be described by the following equation:

$$\frac{\partial \Xi}{\partial A} - h\frac{\partial \Xi}{\partial X} = Dh \tag{1}$$

(Peebles 1980), where we have introduced the following variables to replace the comoving co-ordinate x and scale factor a:

$$\Xi = \ln[(1+\bar{\xi}_2(x,a)], \qquad A = \ln a, \qquad X = \ln x\,. \tag{2}$$

The quantity h is defined through the relation $v = -h\dot{a}x$where v is the mean pair velocity at separation x. The characteristic of the same equation represents conservation of pairs during collapse:

$$l^D = x^D(1+\bar{\xi}_2(x,a)); \tag{3}$$

we have introduced a new length scale $l = \langle x^D \rangle^{1/D}$ which represents the typical initial (Lagrangian) length scale from which structures are collapsing at a certain epoch. This suggests that the nonlinear correlation function can be expressed as a function of the initial (linear) correlation function, evaluated at a different length scale to that at which the final function is given, i.e. $\xi_2(x,a) = F_n[\xi_2(l,a)]$, where the subscript n indicates that the function F is permitted also to depend on the initial spectral index. Since in scale-free gravitational clustering all the one point moments of the distribution depend on epoch a and length scale l only through the volume averaged correlation function $\bar{\xi}_2(x,a)$, evolution of any statistical property can be expressed in terms of such a non-local mapping. For example, the probability distribution function (pdf), $P_\delta(x,a)$, of the density field (δ) can be expressed as a function of initial pdf at length scale l, $P_\delta(x,a) = \mathcal{F}_n[P_\delta(l,a)]$; δ acts as a parameter in such a scaling. Higher order moments of P_δ (which are volume-averaged higher-order correlation functions) will also exhibit scaling properties due to the hierarchical nature of the clustering although knowledge of all the higher order correlation functions does not necessarily specify pdf uniquely (cf. Coles & Jones 1991)

Simulations suggest that gravitational clustering evolves through three different phases. The early phase of clustering, during which the fluctuations are small, can be studied using perturbative methods. The evolution of higher order correlation functions in this epoch is dominated by precollapse evolution of rare events which can be described by a spherical collapse model (Bernardeau 1992, 1994). Scaling takes a very simple form in this case, with $\bar{\xi}_2(x,a) \propto [\bar{\xi}_2(l,a)]$ where x and l are not very different. Consequently h increases almost linearly with $\bar{\xi}_2(x,a)$ in this epoch.

The intermediate regime can be characterized by turnaround and collapse of these very high density peaks, leading to a scaling described by $\bar{\xi}_2(x,a) \propto [\bar{\xi}_2(l,a)]^D$. Hence in any dimension D, this relation has a slope exactly equal to the slope of background space. One of the motivation of this paper is to check this prediction against N-body simulations in 2D. In 3D it has already been studied and shown to follow the predicted scaling (Munshi & Padmanabhan 1997). In this regime, h reaches a maximum value of 2 at turn around before decreasing in highly nonlinear regime when virialised structures start to collapse.

It is generally believed that collapsed haloes are no longer participating in the global expansion and therefore that the average peculiar pair velocity counterbalances the Hubble expansion. This would mean that h attains a asymptotic value of 1 in the final regime of strong nonlinearity. Such reasoning leads to the "stable clustering" *ansatz* which will produce a scaling $\bar{\xi}_2(x,a) \propto [\bar{\xi}_2(l,a)]^{D/2}$. So if stable clustering is a good approximation, the slope of the scaling function is again determined only by dimension of the background space.

Scaling of the higher-order correlations can be analysed by studying the evolution of the so called S_N parameters. To summarize in general we have following forms of higher order correlation functions in the *perturbative*, *intermediate* and *nonlinear* regimes discussed above:

$$\begin{aligned}
\bar{\xi}_N^{pert}(x,a) &= S_N^{tree}\bar{\xi}_{2,lin}^{N-1}(l,a); \\
\bar{\xi}_N^{int}(x,a) &= A^{N-1}S_N^{int}\bar{\xi}_{2,lin}^{D(N-1)}(l,a); \\
\bar{\xi}_N^{non}(x,a) &= B^{N-1}S_N^{non}\bar{\xi}_{2,lin}^{D(N-1)/2}(l,a).
\end{aligned} \tag{4}$$

In these equations A and B are constants determined from the scaling of two point correlation function; the S_N parameters are defined to be the dimensionless ratios of higher order correlation functions with suitably raised powers of two point correlation functions $S_N = \xi_N(x,a)/\xi_2(x,a)^{N-1}$ at same scale, and by definition $S_1 = S_2 = 1$. For the purposes of this paper we are only interested in the slopes of the inferred scaling relations, so we use the quantities S_N as floating parameters in different regimes and use only the slopes in the following analysis.

Using these scaling relations (4), it is easy to construct the complete dependence of the two-point correlation functions on spatial and temporal coordinate i.e. x and a in different regime.

$$\bar{\xi}_2(x,a) = \bar{\xi}_0 a^{2Dh/[2+h(n+D)]}x^{-Dh(n+D)/[2+h(n+D)]}, \tag{5}$$

where n is the spectral index of initial scale free power spectrum. Note that equation (5) is only a solution of equation (1) for $\bar{\xi}_2(x,a) \gg 1$, i.e. in the highly nonlinear regime. This expression makes it clear that assumption of stable clustering leads to a final correlation function which depends explicitly on initial power spectrum. On the other hand one might expect that highly nonlinear gravitational physics might lead to correlation functions that do not depend explicitly on the initial power spectrum. This would translate into a behaviour of h of the form $h(n+D) = K$, where K is a constant. Note that the mean pair velocity, however, is still governed by a dependence on the initial power-spectrum in this case. It is interesting to note that if stable clustering is a good approximation, in 2D the slope of the scaling function in this limit should be the same as it is in the linear regime.

Although several studies have been carried out to check stable clustering ansatz (e.g. Jain, Mo & White 1995; Jain 1997), such studies are affected by limited dynamical range

available for 3D N-body simulations. Since 2D provides much wider dynamic range it is much easier to study extreme nonlinear regime with very high values of $\bar{\xi}_2(x, a)$. It is also worth mentioning that most of the earlier studies have been performed using P^3M codes and it is not very clear if sub-grid resolution in such simulations produce different results compared to more robust PM simulations (Splinter et al. 1997).

3 COMPARISON WITH SIMULATION RESULTS

We have tested these ideas by examining the results of 2D N-body simulations of gravitational clustering, based on the PM algorithm (cf. Beacom et al. 1991). We have run simulations of a variety of realisations of initial power-law spectra, with $P(k) \propto k^n$ with random-phase initial conditions generated in the standard manner. (Recall that a power-law of index n in 2D corresponds to a 3D spectral index of $n-1$.)

The variance at a fixed scale x shows a nonlocal scaling, and it also induces scaling in the volume average of higher-order correlations functions due to hierarchal nature of N-point correlation functions. We can extract moments corresponding to different x and a, $\bar{\xi}_N(x, a)$ from a series of different timesteps of the evolution of a single set of initial data, in much the same way as was done by Jain, Mo & White (1995), and relate them to the the linearly extrapolated variance, $\xi_2(l, a)$, on the appropriate length scale $l = x(1 + \bar{\xi}_2(x, a))^{1/2}$. Determination of higher order correlations are non-trivial due to the presence of final volume effect. We use methods based on factorial moments of the counts-in-cells to determine volume averages of higher order correlation functions; details of the correction procedure are explained by Munshi et al. (1997).

The results obtained by this approach can be used to populate a locus in the plane of $\bar{\xi}_N(x, a)$ against $\bar{\xi}_2(l, a)$ as shown in Figure 1 for $N = 2$, 3 and 4. If scaling holds, the locus should be well defined for results obtained from different evolutionary stages. Comparison of the theoretical slopes at different regimes of gravitational clustering with our simulations show that the scaling arguments presented in Section 2 do indeed work fairly well for all the initial spectra considered, lending further credence to the physical arguments from which they were developed.

It is also clear that agreement with theory is better for models with less power on small scales, the intermediate slope in case of $n = 2$ spectrum shows pronounced deviation from its predicted value 2. This is not altogether surprising, as this model displays very strong clustering on small scales. Since the scaling argument is based on spherical collapse, which might be disrupted by strong sub-clustering, one might have anticipated a better match of results for spectra with less power on small scales. A similar breakdown might be anticipated for results obtained from perturbation theory, due to the presence of large amounts of small scale power (Munshi et al. 1997).

Determination of higher- and higher-order correlations from simulation data becomes increasingly difficult due to their sensitivity to various spurious effects, such as the discrete nature and finite size of simulations. As described in Section 2, the presence of nonlocal scaling in the two-point correlation functions results in a scaling of the entire pdf for scale free simulations. Scaling behaviour in the pdf is much easier to study than its higher order moments. In Figure 1, we present results for the pdf evaluated at an overdensity $\delta = 5$. Studying such nonlocal scaling might help developing a complete theory of evolution of pdf and its higher order moments in different regimes (Colombi et al. 1996), without having to worry about the errors in estimating the high-order correlations from relatively small simulation volumes.

The other issue we sought to address is whether the stable clustering limit is indeed reached during these simulations. However, the results displayed in Figure 1 do not go very far into the strongly non-linear regime. The extremely high resolution of our 2D simulations allows us to calculate much higher variances than can be displayed on Figure 1 without compressing the intermediate regime too much. In fact we find that, although the early stages of the strongly non-linear regime do seem to be well described by the stable clustering ansatz, the curves do show some deviations at very high values of $\bar{\xi}_2$. Such a behaviour might be expected if the simulation were dominated by resolution problems, but we think this is unlikely to be the case here, because the scale concerned is still above the resolution length of the simulations. We also note that the scaling observed has the same form for each evolutionary epoch in the simulations. In the 3D study by Jain, Mo & White (1995) it was found that resolution effects resulted in a deviation from the stable clustering form at different values of $\bar{\xi}_2(x, a)$ for different evolutionary stages (recall that the curves shown in Figure 1 are constructed by co-adding results from different scales and simulation epochs). We do not find this to be the case and consequently argue that the departure from stable clustering we see is not a result of the same resolution problems they found. Moreover, we find that asymptotic power index for correlation functions at very high values of $\bar{\xi}_2$ becomes roughly independent of the slope of the initial power spectrum as has been suggested by Padmanabhan (1996); see also Saslaw (1980). This would imply that the parameter h can expressed as a function of initial power index n according to

$$h = \frac{K}{n+2}, \qquad (6)$$

where K is a constant. Figure 2 shows the behaviour of the measured h as a function of $n + 2$ for these simulations. It does indeed appear to be roughly satisfied although there is significant scatter in the plot. Notice that $h = 1$, independent of n, if stable clustering holds. We feel that the results displayed in Figure 2 are unlikely to represent a resolution effect, because the transition from the initial phase of stable clustering to this later one is extremely smooth and noise-free. Other claims to have detected stable clustering are much easier to interpret in terms of the smaller dynamical range of available 3D simulations (Jain, Mo & White 1995; Jain 1997). For example, Jain (1997) shows how h moves towards unity as $\bar{\xi}_2(x, a)$ increases but his plots cut off before there is any evidence of a period where h is constant at a value of unity. In our simulations, h carries on decreasing below $h = 1$ and shows no sign of asymptoteing to this value. We have used the slope of scaling curves to measure the values of h, other possibility include measuring directly the mean pair velocity. As mentioned in Jain (1997),

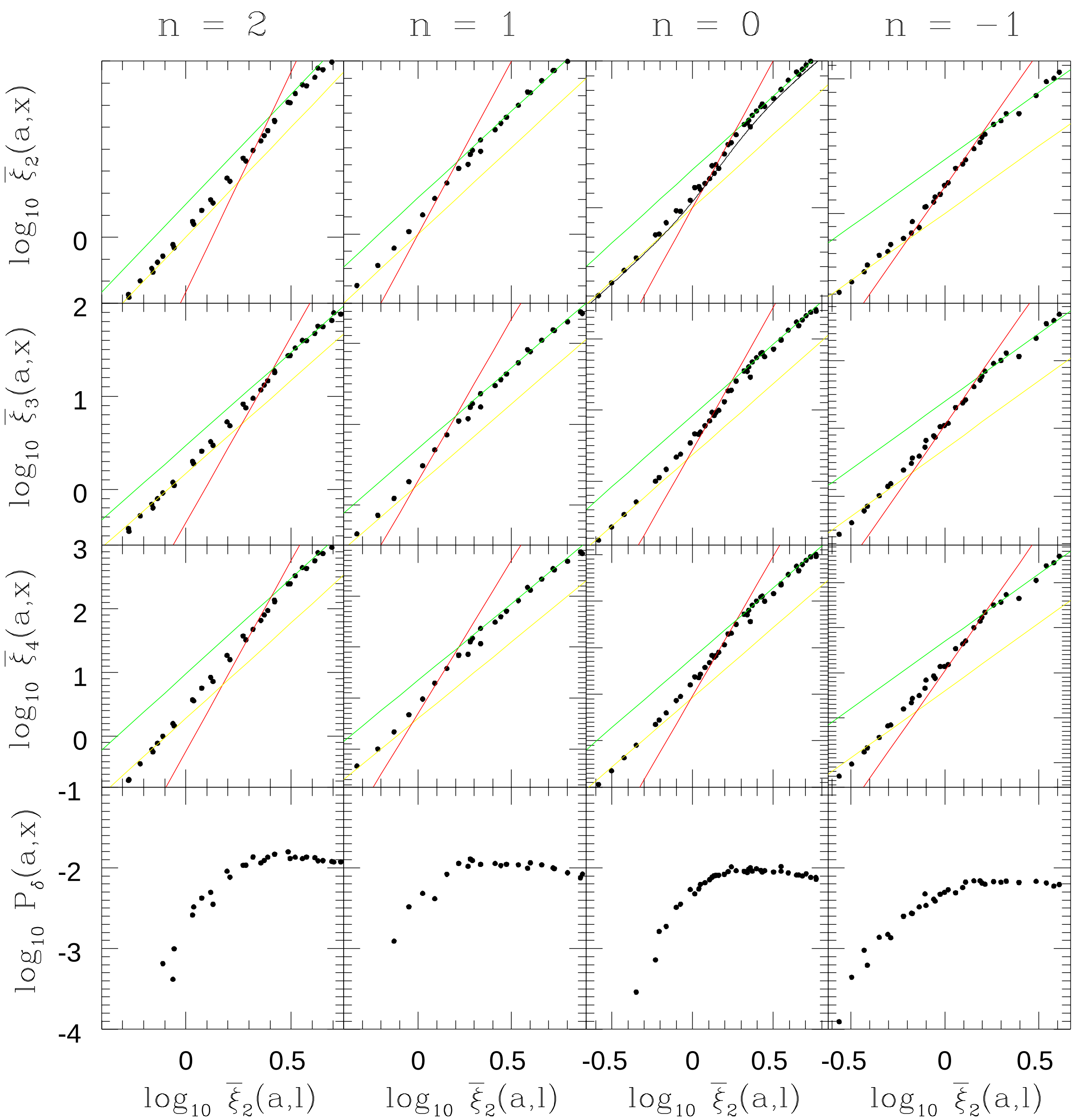

Figure 1. The volume-averaged N–point correlation functions $\bar{\xi}_N(x, a)$ are plotted against the linear two-point function $\bar{\xi}_2(l, a)$ for various power law spectra. Straight lines in different panels corresponds to slopes $(N-1)$, $2(N-1)$ and $(N-1)$ for the perturbative, intermediate and highly nonlinear regimes, respectively. Dots represent 2D N-Body simulation data. The lowest panel shows scaling in the pdf for $\delta = 5$.

the mean pair velocity displays a large scatter around its mean value and a large number of realizations are needed to reduce this dispersion. He used differential form of pair conservation equations to evaluate h from simulation data, equivalently one can use the scaling function directly in its integral form to find out the slope and hence the value of h.

4 CONCLUSIONS

We have addressed two main issues in this paper. The first is the extent to which 2D gravitational clustering can be as well described by simple scaling models as the 3D case appears to be. More importantly, we also wished to determine whether the quantitative scaling behaviour in 2D is as expected on

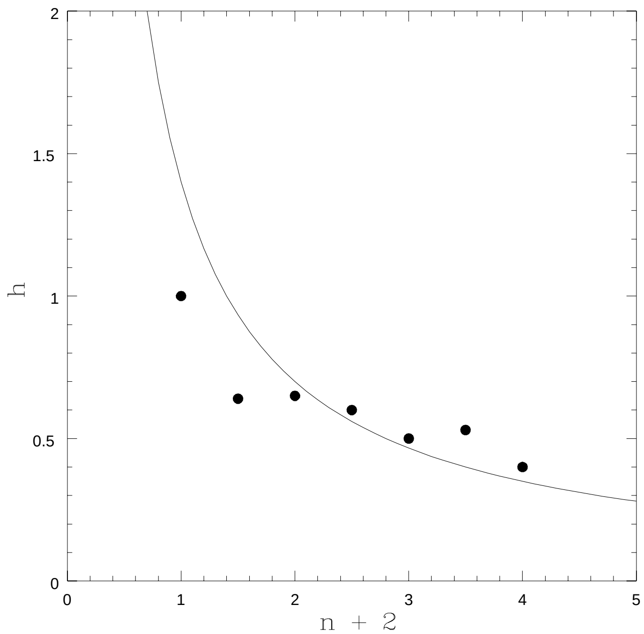

Figure 2. The value of h extracted from the simulations is plotted as a function of $n+2$. The solid curve represents the relationship $h(n+2) = 1.4$, discussed in the text.

the basis of radial collapse arguments. The answer to both of these questions seems to be "yes", suggesting that the scaling arguments used are physically well-motivated and provide a very good description of the simulation data.

The second issue we investigated was whether we could infer from the behaviour of these simulations in the strongly non-linear regime that a regime of stable clustering is reached. The answer to this appears to be "no". Although there is an initial strongly non-linear period of stable clustering, there appears to be a significant departure from this behaviour at very small scales. Of course, it is possible that at even higher clustering amplitudes still, h starts increasing and then settles down at a unit value. All we can say is that there is no evidence from our study that this happens and consequently that if stable clustering ever applies, it can only do so on exceedingly small scales.

We hope to investigate this second issue further in related studies. For example, a stability analysis of the coupled integro-differential equations which govern the behaviour of evolution of correlation functions around self-similar solution in the highly nonlinear regime can also provide important insight into their asymptotic behaviour on small scales. A recent study by Yano & Gouda (1997) demonstrates that power law index of the two point correlation function predicted by stable clustering ansatz is not special in terms of stability of the solution. It is also worth mentioning that the slope of the two-point correlation function can be related to the form of the halo profile at very small radius. This can be done if we assume that, at small length scales, the dominant contribution to the correlation functions comes from points within the same halo. A correlation function which becomes independent of the initial power spectrum at very small scales would lead to halo profiles independent of initial conditions at small radius. Such a direct study of halo properties is, in some sense, more fundamental than the scaling of the correlation functions because while the correlation function can be calculated if halo density profiles are known, the converse is not necessarily the case. There has been considerable interest in recent years devoted to this question in 3D (Navarro et al. 1995; Cole & Lacey 1996; Navarro et al. 1996; Syer & White 1997). We intend to complement these studies with a related investigation in 2D, which will again provide greater dynamical range with which to answer such questions.

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